Lesson 4. Solutions of First-Order Linear Dynamical Systems

- Consider a first order linear DS

$$A_{n+1} = sA_n + b \quad n = 0, 1, 2, \ldots \qquad (\star)$$

Case 1. $s \neq 1$

- If $s \neq 1$, then the general solution to $(\star)$ is

- If A_0 is specified with an IC, then we must have:

- Therefore, if $s \neq 1$, the particular solution to $(\star)$ that satisfies the IC is

Case 2. $s = 1$

- If $s = 1$, then the general solution to $(\star)$ is

- If A_0 is specified with an IC, then we must have:

- Therefore, if $s \neq 1$, the particular solution to $(\star)$ that satisfies the IC is

Example 1. Suppose we have a bank account earning 5% interest, compounded annually. We deposit $500 initially, and also deposit $100 at the end of each year.

    a. Write a DS that models this setting.
    b. Write the IC for this setting.
    c. Find the particular solution for the DS you wrote in part a that satisfies the IC you wrote in part b.
    d. How much do we have after 10 years?

Suppose now that interest is compounded monthly instead, and we deposit $$\frac{100}{12}$$ at the end of each month.

    e. Write a DS that models this setting.
    f. Write the IC for this setting.
    g. Find the particular solution for the DS you wrote in part e that satisfies the IC you wrote in part f.
    h. How much do we have after 10 years?
    i. Compare the amount in our account with annual compounding vs. monthly compounding.

Example 2. Suppose we win the lottery. We have two options:

   Option 1. $500,000 now
   Option 2. $50,000 in 20 annual payments, with the first payment given now

Assume that whenever we get a payment, we put it in an account earning interest at an annual rate of r, compounded annually. We want to know which payment option leaves us with the most money at the time we get the last of the 20 payments.

  a. Write a DS that models Option 1.
  b. Write the IC for Option 1.
  c. Find the particular solution for the DS you wrote in part a that satisfies the IC you wrote in a.

  d. Write a DS that models Option 2.
  e. Write the IC for Option 2.
  f. Find the particular solution for the DS you wrote in part a that satisfies the IC you wrote in d.

g. Which payment option is better when $r = 0.05$?
h. Which payment option is better when $r = 0.10$?